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Minimum Wages in General Equilibrium

by

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A thesis submitted in partial fulfillment
of the requirements for the
Degree of Bachelor of Arts with Honors
in Economics

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Minimum Wages in General Equilibrium

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Minimum Wages in General Equilibrium*

Abstract

Classical economics is thought to predict that minimum wage policy reduces the employment levels of low-skilled individuals. However, in a famous paper, Card and Krueger (1993) find ambiguous and possibly positive employment impacts in New Jersey's fast food sector following a minimum wage increase in the state. This result has been a point of contention and puzzlement in the literature for decades. This paper adds to the literature through providing theoretical motivation for the results of Card and Krueger (1993) with a two-good, two-consumer model with heterogeneous preferences and production. This results in the introduction of a minimum wage causing different general equilibrium effects in each sector. The equilibrium in this model reproduces some of the empirical results observed by Card and Krueger (1993) among others. This analysis moves to a more long run oriented analysis inspired by the dynamic stochastic production model introduced by Hopenhayn (1992). This thesis uses these models to gain a better understanding of the expected short and long-term consequences of minimum wage policy.

1 Introduction

Minimum wage policy remains a heavily contested economic and political issue in the United States. Events such as the unresolved debate over the Minimum Wage Fairness Act in April 2014 and the raising of the minimum wage to $15 per hour in Seattle demonstrate that the issue is unlikely to be resolved on either a national or a state level in the near future. The empirical literature on the efficiency of the minimum wage has been similarly conflicted, with various papers reporting negative, ambiguous, or at times positive employment effects as a result of minimum wages (e.g. Card and

*Sincere thanks are due to Professor Rolleigh, who has been incredibly patient with me as I worked through this project. Professor Love and Professor Watson have also provided numerous helpful comments for which I am appreciative. Finally, I would like to thank the entire economics department of Williams College for a wonderful four years of education.

Krueger (1993) and Neumark and Wascher (2006)). However, many of the economists tacitly agree with the statement that theory unambiguously predicts minimum wages to decrease low-skilled employment across all sectors. This attitude may have been inspired by a partial equilibrium analysis from Stigler (1946) and has proved difficult to shake off or challenge.

This paper challenges this perspective through demonstrating that the theoretical predictions of minimum wage impacts may not be so straightforward. To do this, I construct a model demonstrating the competing forces that may shift low-skilled employment ambiguously upon a minimum wage shock in a society where consumers of different skill levels value goods differently. The intuition behind this is that minimum wage shocks may increase the demand in sectors relatively preferred by low-skill workers enough to offset decreased labor demand effects following the wage changes. The analysis is extended as I move to a more advanced, dynamic production model introduced by Hopenhayn (1992) that factors in random production shocks. I then calibrate the model with values suitable for the United States to gain a clearer understanding of predictions for the efficiency of minimum wage policy within the U.S. The dynamic model predicts minimum wages to decrease total output and labor employment in the economy while increasing average output, indicating a "survival of the fittest" dynamic that leads to only high-producing firms staying in business. Low-skill labor employment in particular falls by roughly three percent in response to a 20 percent minimum wage increase, consistent with past empirical estimates. Thus, from a stationary equilibrium analysis, the long-run impacts of minimum wages may not be so beneficial for low-skill workers.

Surprisingly little literature in economics has been dedicated to theoretical analyses of minimum wage policy or comparisons of it to the merits of other policies. There has also been little effort to theoretically produce the counterintuitive results of Card and Krueger (1993). My work contributes to the literature through investigating minimum wage impacts in the presence of heterogeneous preferences in competitive markets. Through this, I provide new theoretical motivations behind the results that Card and Krueger reach. My paper is also the first to diversify labor in Hopenhayn's workhorse model. It is the goal of this paper to contribute to the evaluation of a contemporary and important economic issue.

Section two reviews of minimum wage policy. Section three will present a simple model that demonstrates the possibility of conflicting forces pushing low-skill employment after a minimum wage shift. Section four presents results of this model, section five goes over the construction and results of the dynamic programming model, and section six concludes.

2 Empirical and Theoretical Background

2.1 The Minimum Wage Debate

A seminal work on minimum wage theory is Stigler (1948), who argues that minimum wages will lead to decreased employment levels and that there are more effective measures for helping the poor. Stigler's reasoning is as follows: in competitive markets, an increase in the wage rate will either lead to the dismissal of low-productivity workers or the same workers improving in productivity to become worthy of their increased wage. However, Stigler argues that large productivity boosts are unlikely to occur in large and low-wage markets; thus, decreased employment must follow wage boosts instead. Non-competitive firms may have an optimum wage that can be set, but variability of optimum wages across firms and time would make setting a single perfect minimum wage impossible. Stigler also notes that hourly wage and family income, as well as hourly wage and yearly earnings, may not be strongly correlated, concluding that minimum wages would be an ineffective poverty-targeting tool. Other, more recent literature has come to the same conclusion. As a result, Stigler asserts that a minimum wage raise would unambiguously be an ill-targeted and employment-harming policy.

However, Stigler's conclusions have not always been observed in the real world. In a famous paper, Card and Kruger (1993) investigate the impact of minimum wage increases fast food industries in 1992 in New Jersey while using Pennsylvania (where wages did not increase) as a control. As the minimum wage increase occurred during a recession, the authors believe that improving economic conditions would not have obscured the impact of the wage increase. With the data, Card and Kruger employ a difference-in-differences model comparing the employment levels in Pennsylvania and New Jersey and fail to find any indication of decreased employment levels in New Jersey after the minimum wage increase, nor do they find any smaller fraction of workers working full-time.

Card and Kruger focus their investigation by interviewing fast-food establishments about employment levels – a useful sample due to the amount they employ minimum-wage workers, their homogeneity, and the absence of tips in the business. The large majority of franchises that were interviewed in the first round were again interviewed in the second, and the authors fully account for stores closing. The natural experiment is further made possible by Pennsylvania not having any minimum wage increase. The authors interview 410 fast-food restaurants in March 1992 and 399 in December 1992, roughly 3/4 in New Jersey, the rest in Pennsylvania. Locations were asked about how many workers they employ and what wages they are paid. They then compare the different reported employment rates in the two states and evaluate the impact of the minimum wage increase through a difference-in-differences model, with the dependent variable being employment levels and the key independent variable being different wages in New Jersey over time. Their evaluation confirms the validity of the use of Pennsylvania as a control and finds that employment actually expanded in New Jersey low-wage stores.

Card and Krueger then include a regression-adjusted model that controls for state, franchises of the same chain and the proportional increases of wages necessary to meet the new minimum wage in a given store. The authors also include changes in mean employment as an independent variable. Card and Krueger also divide the two states into regions for even stronger tests and later run more specification tests by excluding various stores or introducing various weights. The authors fail to identify any negative employment effects from the New Jersey minimum wage laws.

Though a landmark and compelling paper, there are many possible concerns with it, as Neumark and Wascher (2006) point out. Among the issues is the lack of lag variables, the use of phone surveys, and, most problematic, the use of Pennsylvania as a control for New Jersey. The latter concern comes from the fact that teenage employment rates in Pennsylvania differed markedly from those observed in New Jersey during the period of the study; which may have made Pennsylvania an improper control.

In a paper related to Card and Krueger (1993) but more narrowly focused, Dube et al focus (2010) on a city-specific minimum wage increase – that in San Francisco in 2004 and 2007. This paper includes both table service and fast food – similar to Card and Kruger, the paper finds no disemployment consequences from the minimum wage increases, although the fast-food and table service establishments responded differently. There is also less measurement error, which gives the authors more confidence in their results. While the primary dependent variables Dube et al examine are average hourly pay, distribution of pay, and employment statistics, they also observe menu prices, employee tenure, health insurance coverage, tips, and employer law compliance. None of these variables change significantly in response to the minimum wage increase either.

However, not all economists find such ambiguous results regarding minimum wages. Neumark and Wascher (2006) provide an extensive summary of papers about the effects of minimum wage increases in the United States and elsewhere, beginning in the 1990s. While some studies find no employment effect or even increased employment following minimum wage increases – a notable example being Card and Kruger (1994) – Neumark and Wascher find that a majority of minimum wage studies they deem credible find negative employment effects overall, especially for teenagers and other low-skill or at-risk groups.

Neumark and Wascher find negative employment effects in several of their own papers, and Neumark noticed a general trend for him and Wascher to find negative employment effects while Card and Krueger to find ambiguous or positive results, perhaps suggesting authorial bias. To remedy this, prior to actually investigating the data, Neumark (2001) and other authors (Card and Krueger being notable exceptions) agree on a pre-specified statistical design. The authors state that design has limited applicability to data due to its rigor making many data sets unusable, but for the data that it can be applied to, it produces results familiar to those reached by Neumark in other papers: that minimum wage increases have disemployment effects for younger, less-skilled workers. The paper also provides a sort of meta-analysis; it collects the elasticities of employment with minimum wages found in various articles. With this, Neumark demonstrates two main points: first, that a large variety of elasticity estimates have been produced, and that certain authors tend to be associated with certain findings (i.e., Card and Krueger find no impacts of minimum wages while Neumark and Wascher find negative impacts)

Card and Krueger also pursue a similar vein of meta-analysis. Card and Kruger recognize that time-series data tends to indicate a 1-3% employment drop for teenagers when the minimum wage is increased by 10 percent. Cross-sectional studies tend to produce different estimate, however, and Card and Kruger seek to evaluate the validity of the time series estimates. Sampling theory predicts a positive relationship between sampling size and t-ratios – however, in their survey of the literature, Card and Kruger find a negative relationship between the two, which they interpret to indicate bias in the literature towards statistically significant results.

Studies that produce statistically significant results tend to be viewed as more important. The presumption that minimum wages should lead to higher unemployment may also result in bias. To test this, Card and Krueger rely on basic sampling theory. More recent studies on minimum wages use more data – in some cases, the number of observations is twice as large, which should lead to a lowering of standard error and an increase of the t-ratio by about 40 percent. Despite time series data not likely being independent, the t-ratio should increase with sample size. In their analysis, Card and Kruger gather 15 studies from as early as 1970 and observe the relationship between the t-ratios within them to the respective degrees of freedom. The dependent variable is the log of the t-ratio for each study, and the key independent variable is the log of the square root of the degrees of freedom in each study. Surprisingly, Card and Krueger find a negative relationship between the two. Additionally, Card and Krueger notice that t-ratios tend to be near to exactly two in many of the studies, which may suggest deliberate manipulation of specifications to produce barely significant results. While Neumark and Wascher also mention that time-series studies are becoming increasingly obsolete due to state-specific minimum wage changes, Card and Krueger's findings indicate that such studies may have never been reliable in the first place.

Some studies compare U.S results to those of other countries. Currie and Fallick (1996) analyze the impact of minimum wage increases on the employment levels of individuals employed at or near the minimum wage prior to the shift through NLSY data. Abowd et al improve (1990, 2000) upon the procedure and undertake a similar analysis of such workers in both the United States and France. On the workforce as a whole, the authors find that the minimum wage increases have mild effects on employment – however, the effects are much more pronounced on the group that is working at or near the minimum wage in the beginning, with workers in France being especially negatively impacted. France serves as a good country to compare to the United States given that its real minimum wages over the years have moved in the opposite direction.

French data is extracted from the "Enquete Emploi" from 1981 to 1989. The survey included sixty thousand households, most of them interviewed in each wave. Meanwhile, the U.S. data are NBER extracts from the CPS between the years of 1981 to 1987. The key regression is a conditional logit analysis of the probability of being employed in period t+1 given ones wage in period t and another important control in the form of age. Wages are divided into four categories: below the starting minimum wage, around it, marginally above it, or well above it. Ultimately, the authors find that lower wages in period t result in lower probabilities of being employed in period t+1, especially for younger workers.

Other studies focus on exclusively international comparisons. Dolado et al (1996) survey several countries and the minimum wage impacts within them throughout Europe, focusing especially on the United Kingdom, the Netherlands, France, and Spain. Minimum wages have seen little change in their size compared to average wages in Europe over the years. The authors first go over the different types of minimum wage structures seen in Europe. The UK has the option of minimum wages being set in certain industries – in the other three countries, the statutory minimum is set by the government overall. Their discussion is largely non-technical, and on the whole they report that the effects of minimum wages are exaggerated – while some effect for youth employment may be observable, the institution of minimum wage laws result in little change overall for the involved economies.

Some of the theoretical underpinnings of the authors argument revolve around monopsonistic theory; in essence, a few industries may employ the bulk of labor of a certain skill level and be unable to move away from using it as a result. The authors then discuss in-depth the four aforementioned countries - overall, any evidence for negative employment effects stemming from minimum wages is slight.

France instituted the Salaire Minimum Interprofessionnel de Croissance, a guaranteed minimum wage that rose when inflation was sufficiently high. The authors divide the labor market into several levels in order to isolate the effects of the minimum wage on the part of the market most likely to be affected by it. The primary regression is simply wage changes on employment levels – some negative effects are found, but there are several possible explanations outside the increasing minimum wage.

In the Netherlands, a series of wage-based minimum wages were implemented, starting in 1968. The progression of extending minimum wages to younger and younger workers provided a natural experiment of sorts, and the authors look at employment effects on the youngest workers over time, finding no compelling evidence of negative employment effects.

Spain's national minimum wage has changed especially little compared to the average wage of the country over time, making it difficult to estimate employment impacts. Similar to before, the authors split the economy into several sectors and look at those most likely to be impacted. The authors find negative employment impacts for youth, but ambiguous or possibly positive employment impacts overall.

In the United Kingdom, the authors take advantage of the industry-specific wages set by the UK Wage Councils to investigate minimum wage impacts. Simple, but surprisingly robust regressions of wages on employment levels indicate that industries with higher minimum wages grew faster. At the very least, no negative employment impact is detected.

2.2 Theoretical Literature on Minimum Wages

Stigler's conclusions have been contested on a theoretical level as well. Rebitzer and Taylor (1995) investigate the consequences of minimum wages when one relaxes the assumption that productivity is not wage-dependent. When efficiency wages are a factor, firms can behave in a monopsonistic manner. Rebitzer and Taylor characterize employees as being capable of working at two different intensities and employers being able to only imperfectly monitor their employees' effort. In the model, a higher wage can inspire higher effort from the employees, thus boosting firm productivity, contrary to Stiglers prior reasoning. From this, the authors find that minimum wages can increase employment in both the short and long run.

Many authors introduce minimum wages in general equilibrium models and produce interesting results. An especially relevant contribution is Aaronson, French and Sorkin (2013), who present a theoretical analysis of minimum wages that focuses on firm entry and exit in response to minimum wage shocks. In addition to finding ambiguous employment impacts following minimum wage implementation, the authors find empirically that both market entry and exit rise in response to minimum wage changes. This is a surprising result, as most canonical models predict increased exit and reduced entry. The authors then develop a Putty-Clay production model – one in which potential market entrants can select from a rich variety of capital combinations but must commit to sticking with a capital combination upon entering the market – that explains this phenomenon. The firms that are unable to adapt and operate profitably in response to the minimum wage hikes leave the market, and new entrants who are better able to exploit the change enter. Thus, minimum wages may drive up both market entry and exit, which may make predicting the employment and welfare consequences of the policy more difficult.

Bhaskar and To (1999) develop a market framework that provides a theoretical justification for the empirical results derived by Card and Krueger (1993). The authors argue that the fast-food industry may be described as a monopsonistic competition, with subtly different firms competing for the same workers. The monopsonistic nature of the market allows for a minimum wage increase to raise employment at the firm level, but the competitive nature of the industry also leads to minimum wages spurring greater market exit rates. As a result, the net employment effects of a minimum wage hike are ambiguous. Welfare implications, the authors find, may depend on the reservation wages of workers.

Both Flinn (2005) and Ahn and Arcidiacono (2003) investigate the impacts of minimum wages in economies where firms and workers search and match themselves to one another. Like in Bhaskar and To (1999), Flinn (2005) specifies his economy as a monopsonistic one and produces the unusual result that minimum wages can increase overall welfare in the economy. The intuition behind the conclusion is that the minimum wage can make wage-bargaining processes quicker and more efficient, thus reducing searching frictions in the economy. Furthermore, Ahn and Arcidiacono (2003) report that, in a searching and matching economy, minimum wages can incentivize new potential workers to enter the economy and look for a job, thus increasing low-skill employment.

Finally, Cahuc and Michel (1996), show possible beneficial effects of minimum wage legislation from a different angle. The authors assume that human capital accumulation has positive externalities. They then construct an overlapping generations model wherein each agent lives for three periods and chooses to be either skilled or unskilled in the first period. Choosing to be either skilled or unskilled comes with it a set of costs, future wages, and externalities. The authors demonstrate that minimum wage laws, through decreasing demand for unskilled labor, may increase the incentive for individuals to accumulate human capital. In their model, this effect outweighs the possible disincentive for skill acquisition that may arise from the wages of low and high-skill jobs becoming more similar. Through decreasing the demand for low-skill labor, the minimum wage shift increases relative demand for high-skill labor, which may compel more individuals to choose to be skilled in the first period of their lives. This, in turn, can strengthen the long-run production of the economy, so that it is possible that a decrease in the minimum wage may decrease overall welfare.

My work makes the following contributions to the literature on minimum wages. Other papers have focused less on the possibility of individuals having different demand functions for different goods based on their income level. The presence of heterogeneous preferences allows for calibrations to observed spending shares by workers of different skill levels. My work will also differ from Bhaskar and To through demonstrating ambiguous minimum wage effects in competitive market settings. Finally, my work will be the first to adapt the model set forth by Hopenhayn (1992) to allow for minimum wage changes. Unlike Aaronson, French and Sorkin's model, firms can adjust production levels and input ratios after entering the market, allowing for firms that have more flexibility when choosing to enter or stay in the market. For all these reasons, my project will add to the literature of minimum wage theory.

3 The Two-Sector Model

3.1 Overview

The theoretical literature on minimum wages has done little to investigate economies with consumers that have heterogeneous preferences correlated with their skill level. Such a feature may importantly affect how minimum wages impact the economy in general equilibrium. In this section, I model a two-good market in which, with certain parameter values, either a progressive transfer from a high-skilled to a lower-skilled individual or the establishment of a minimum wage law results in greater labor supply in the market for a cheap good due to increased demand for said good. If consumption preferences are correlated with income, then progressive income policy in the form of a minimum wage shift may raise demand for goods produced by low-skill, low-income laborers. If the demand shift is great enough, it could potentially outweigh labor demand decreases caused by higher wages.

3.1.1 Fast Food Industry Statistics

Surprisingly, there has been little attempt in the literature to theoretically reproduce the fundamental result observed in Card and Krueger (1993) in competitive settings. The result is simple but counterintuitive: upon the imposition of a new minimum wage policy, a sector that is highly reliant on low-skill, frequently minimum wage-earning labor, somehow does not decrease employment. While it is often claimed that classical economics necessarily predicts negative employment impacts as a consequence of minimum wages, this statement is incorrect. Economies where agents have heterogeneous preferences that are correlated with their skill and income levels may produce interesting results when minimum wage policies are introduced in them. The fast food economy in the United States may well be an example of such an economy: approximately 60 percent of food service workers in the United States work at minimum wage levels (BLS); this number may be even higher for those specifically working in fast food. However, the large majority of minimum wage workers are not employed in the fast food industry - this is what allows the increase in minimum wage to drive up demand for fast food enough that firms need to employ more low-skilled labor to satisfy market demand for fast food. Additionally, individuals of lower income levels in the United States likely spend a higher proportion of their income on cheap or fast food than those of higher levels - the PSID reports that individuals across all income groups spend roughly 5% of income on "going out" to eat. Data on how frequently individuals go out to eat at fast-food venues specifically is unavailable, but it is reasonable to assume that the food that lower-income groups purchase when going out is relatively cheaper than the food purchased by their wealthier counterparts. From this, reasonable estimates would be that low-income individuals spend roughly 4 percent of their income on fast food, while wealthier individuals spend between 1 and 2 percent. Thus, a model with heterogeneous preferences in the fast food market can be justified by real-world data, and a calibration of the parameters can be derived from statistics.

3.2 Specification

Assume that the economy has two representative agents for high-skilled and low-skilled laborers. There are two goods in the economy: Fast Food (F), a cheap good, and Everything Else (E), a more expensive good that both types of consumers value more highly. Two different representative firms produce the two goods. I will be focusing primarily on the producer of F.

Assume that both individuals have Cobb-Douglas utility functions over F, E and leisure, R:

$$u_l(F, E, R) = F^{\alpha_1} E^{\alpha_2} R^{(1-\alpha_1-\alpha_2)};$$

$$u_h(F, E, R) = F^{\beta_1} E^{\beta_2} R^{(1-\beta_1-\beta_2)}$$

with $\alpha_1 > \beta_1$, $\alpha_2 > \alpha_1$ and $\beta_2 > \beta_1$. Thus, the richer individual, being more used to luxury, derives relatively less utility from the cheap good than the poorer individual. The preferences for F are different in order to motivate the main point of the analysis in this section: potential ambiguity of minimum impacts stemming from heterogeneous preferences by income. As the poorer individual spends a larger portion of his income on F, a progressive income transfer from the high-skill to the low-skill agent will result in increased market demand for F. It is also reasonable to assume that the consumers will spend a larger share of their income on everything else than fast food. This assumption of asymmetric demand for consumers of different income levels will prove important soon. Consumer 1 can dedicate between 0 and T_l hours of labor to the economy; consumer 2 can dedicate between 0 and T_h hours. Initially, $T_h = T_l = 1$. Consumer 1 is paid w_l for a full unit of labor, and consumer 2 is paid w_h .The price of the first good is p_1 and the price of the second good is p_2, normalized to 1, with $p_2 > p_1$. With constant returns to scale in production functions and free entry, there must be zero profits in the economy. Due to this, there is no question of firm ownership that has to be resolved. Marshallian demand functions for consumer 1 will be

$$x_f^l = \frac{\alpha_1 w_l T_l}{p_f};$$

$$x_e^l = \frac{\alpha_2 w_l T_l}{p_e}.$$

Demand functions for consumer 2 are symmetric. Thus, market demands for goods F and E are

$$F = \frac{\alpha_1 T_l(w_l) + \beta_1 T_h(w_h)}{p_f};$$

$$E = \frac{\alpha_2 T_l(w_l) + \beta_2 T_h(w_h)}{p_e}.$$

We can also derive labor supply equations for the two consumers as well. Low and high-skill labor supply in the market is given by

$$L = T_l - \frac{(1-\alpha_1-\alpha_2)T_l(w_l)}{w_l}.$$

High-skill labor supply is defined by

$$H = T_h - \frac{(1-\beta_1-\beta_2)T_h(w_h)}{w_h}.$$

I now move to the production side and can begin defining market equilibrium. There are two representative producers in the economy, both with constant returns to scale constant elasticity of substitution production functions that differ in their factor shares and elasticity coefficients:

$$P_f(L,H) = Y_f = (\delta L^{\rho_f} + (1-\delta)H^{\rho_f})^{1/\rho_f};$$

$$P_e(L,H) = Y_e = (\phi L^{\rho_e} + (1-\phi)H^{\rho_e})^{1/\rho_e}.$$

Where Y_f and Y_e are total output levels for F and E. Low and high-skill labor are substitutable production inputs that may differ in their productivity levels. In my calibration, a baseline setting for ρ_e is $\rho_e = -1$ following Young's (2011) estimates of aggregate market elasticity. The assumption of zero profits also allows us to derive expressions for prices. The price of either good must be equivalent to the cost of producing a single unit of it, assuming optimal proportions of low and high-skill labor. For the firm that produces F, one can find optimal amounts of inputs to produce a single unit of output to be:

$$H_f^* = (w_h\delta)^{1/\rho_f - 1}\left(\delta(w_l(1-\delta))^{(\rho_f/\rho_f - 1)} + (1-\delta)(w_h\delta)^{(\rho_f/\rho_f - 1)}\right)^{-1/\rho_f};$$

$$L_f^* = (w_l(1-\delta))^{1/\rho_e - 1}\left(\delta(w_l(1-\delta))^{(\rho_e/\rho_e - 1)} + (1-\delta)(w_h\delta)^{(\rho_e/\rho_e - 1)}\right)^{-1/\rho_e}.$$

with symmetric expressions for the firm for E. Thus, the prices of goods F and E must be

$$p_f = w_h H_f^* + w_l L_f^*$$

and

$$p_e = w_h H_e^* + w_l L_e^*.$$

With prices defined, market-clearing conditions for the F and E markets can be defined as well:

$$F = \frac{\alpha_1 T_l(w_l) + \beta_1 T_h(w_h)}{p_1} = Y_e;$$

$$E = \frac{\alpha_2 T_l(w_l) + \beta_2 T_h(w_h)}{p_2} = Y_f.$$

The next step is to define market-clearing conditions for labor markets. To determine how much labor the firms will want at a given output level Y_i, I use the first-order conditions. For the F firm, at any given output level, the firm will want:

$$L = Y_f(w_l(1-\delta))^{1/\rho_f - 1}\left(\delta(w_l(1-\delta))^{(\rho_f/\rho_f - 1)} + (1-\delta)(w_h\delta)^{(\rho_f/\rho_f - 1)}\right)^{-1/\rho_f}$$

and

$$H = Y_f(w_h\delta)^{1/\rho_e - 1}\left(\delta(w_l(1-\delta))^{(\rho_e/\rho_e - 1)} + (1-\delta)(w_h\delta)^{(\rho_e/\rho_e - 1)}\right)^{-1/\rho_e}$$

for quantities of low and high-skill labor, with symmetric expressions for the producer of E. Thus, labor markets clearing entails:

$$\begin{aligned} L = T_l - (1+\alpha_1+\alpha_2)T_l = \\ Y_f(w_l(1-\delta))^{1/\rho_f - 1}\left(\delta(w_l(1-\delta))^{(\rho_f/\rho_f - 1)} + (1-\delta)(w_h\delta)^{(\rho_f/\rho_f - 1)}\right)^{-1/\rho_f} + \\ Y_e(w_l(1-\phi))^{1/\rho_e - 1}\left(\phi(w_l(1-\phi))^{(\rho_e/\rho_e - 1)} + (1-\phi)(w_h\phi)^{(\rho_e/\rho_e - 1)}\right)^{-1/\rho_e} \end{aligned}$$

and

$$L = T_h - (1+\beta_1+\beta_2)T_h = \\ Y_f(w_h\delta)^{1/\rho_f-1}\left(\delta(w_l(1-\delta))^{(\rho_f/\rho_f-1)} + (1-\delta)(w_h\delta)^{(\rho_f/\rho_f-1)}\right)^{-1/\rho_f} + \\ Y_e(w_h\phi)^{1/\rho_e-1}\left(\phi(w_l(1-\phi))^{(\rho_e/\rho_e-1)} + (1-\phi)(w_h\phi)^{(\rho_e/\rho_e-1)}\right)^{-1/\rho_e}.$$

3.3 Equilibrium

General equilibrium can be defined by the vector of outputs and prices $\{Y_f, Y_e, w_l, w_h\}$ that solves the four market-clearing equations. These equations can then be input into a programming language and then solved using a nonlinear equation solver. The wage of the high-skilled laborers is normalized to 1 to ensure that a unique equilibrium exists. Minimum wages are introduced artificially through decreasing T_l, their endowment of hours to contribute to the economy. Directly imposing the wage constraint results in the multiple nonlinear equation solver I use to solve the model not working, which leaves me unable to report economy statistics after the minimum wage shift. I use this method instead so that I am capable of reporting welfare and employment changes in the economy after the minimum wage is imposed. The reduction of hours results in a decrease of their labor supply and an increase in their wage rate – I treat the new wage rate as the imposed minimum wage. One must be careful when evaluating welfare impacts with this method, as one must ensure that the hours that are temporarily taken away are given back to the consumer as leisure that they must consume. This method does not exactly imitate the dynamics of a minimum wage simply being imposed in the economy because an imposition of a minimum wage in the real world does not entail forcibly altering time endowments, but the robustness of the results should not be altered - if anything, this method should understate the results compared to an actual minimum wage shift, as agents are forced to consume more leisure than they would necessarily want to. In the real world, the demand increases for F would be greater in response to a minimum wage shift than in this simulation.

3.4 Limiting Cases

To gain more intuition about the model, it is also helpful to look at the limiting cases in the production function as the ρ_i approach 1 or $-\infty$. As ρ_i approaches 1, the CES production function of interest becomes a linear production function, and as ρ_i approaches $-\infty$, the production function becomes Leontief. Three cases are worth investigating in how they respond to the imposition of a price floor for low-skill labor: the case where both production functions are Leontief, both are linear, and one of each. In the case where both are Leontief, or when both firms have perfectly inelastic production functions, the inclusion of a minimum wage quickly makes the labor-clearing conditions unsolvable – the reduction in T_l forces some unemployment for the high-skilled laborer. This happens because with the Leontief production functions for F and E:

$$Y_f = \min(\delta L, (1-\delta)H);$$

$$Y_e = \min(\phi L, (1-\phi)H).$$

Market clearing conditions for labor with demand functions held equivalent are simply

$$(\alpha_1 + \alpha_2)T_l = Y_f/\delta + Y_e/\phi;$$

$$(\beta_1 + \beta_2)T_h = Y_f/(1-\delta) + Y_e/(1-\phi).$$

In a simple parameterization where all utility weights and factor shares are set to 1/2, changing T_l results in the sum of the input demands equaling two different things. The cases with both production functions being is also interesting – assuming, for instance, that the new production functions for F and E are:

$$Y_f = \delta_1 L + \delta_2 H;$$

$$Y_e = \phi_1 L + \phi_2 H.$$

Then first-order conditions imply that the producer will only employ low or high-skilled labor based on the relationship of $\frac{w_l}{\delta_1}$ to $\frac{w_h}{\delta_2}$. Whichever one is smaller gives the producer a better "bang for his buck," and because the inputs do not exhibit diminishing returns, the cost-minimizing producer will only employ one input. If $\frac{w_l}{\delta_1} = \frac{w_h}{\delta_2}$, then the producer is indifferent between inputs, but the addition of a minimum wage will immediately make the producer move away entirely from low-skilled labor. Thus, the case with both production functions being linear results in minimum wages having severe disemployment effects for low-skill workers.

Additionally, consider the case where the production function for F is Leontief and the production function for E is linear, starting with $\frac{w_l}{\phi_1} = \frac{w_h}{\phi_2}$ so that the producer of E is indifferent between inputs. The imposition of a minimum wage will immediately result in the producer for E forsaking low-skill labor in the production of E, so the producer of F must employ more low-skilled labor if the market for low-skilled labor is to clear. Cases in which one production function is CES and the other function is Leontief or linear also behave as one would expect them to.

3.5 Calibration

Following Young (2011) and considering high-skill labor to be augmented by capital, I set $\rho_e = -1$, corresponding to an elasticity of substitution of $\frac{1}{2}$ in the everything else sector. For the fast-food industry, ρ_f is allowed to vary, as the elasticity of substitution in the fast-food industry is less well estimated, and it is interesting to observe how the behavior of the equilibrium changes as ρ_f is altered. Assuming that consumers spend roughly half their available time as leisure, I calibrate $\alpha_2 = .46$ and $\beta_2 = .49$. To reflect different shares of income spent on fast food, I set $\alpha_1 = .04$ and $\beta_1 = .01$. As the elasticities of substitution are altered, the production share parameters ϕ and δ are calibrated according to:

$$\delta = \frac{w_l(L_f)^{1/\rho_f}}{w_l(L_f)^{1/\rho_f} + w_h(H_f)^{1/\rho_f}};$$

$$\phi = \frac{w_l(L_e)^{1/\rho_e}}{w_l(L_e)^{1/\rho_e} + w_h(H_e)^{1/\rho_f}}$$

where w_l and w_h are wages for low and high-skilled wages observed in the real world, calibrated to be $w_l = 8$ and $w_h = 25$. L_f is the proportion of labor in the F industry that is low skill and is calibrated to be $L_f = .6$, so $H_f = .4$. For the E sector, I calibrate $L_e = .1$ and $H_e = .9$.

4 Results of the Two-Good Model

4.1 Initial Equilibrium Characteristics

Figure 1 illustrates how the degree of heterogeneity in preferences impact demand for low-skill labor in the fast-food sector. As one can see, increasing the ratio of α_1 to β_1 results in almost linear increases in low-skill labor demand in the F sector. The intuition behind this is simple; I hold β_1 constant and increase α_1, and doing so linearly increases the demand for F in the economy. To meet this demand, the F sector must employ more low-skilled workers. Meanwhile, figure 2 demonstrates increasing the elasticity of substitution in the production function also increases the amount of low-skill labor employed in the fast food sector, but the increases follow an S-shaped curve, leveling off as $\rho_f \to 1$. This is also unsurprising – as the firm becomes more capable of switching between certain types of labors, they will choose to employ the cheaper type more. Worth noting is the amount of low-skilled labor employed by the F sector. For reasonable calibrations, the amount demanded is generally around .02 or .03 out of one unit, a very small fraction. Thus, this economy does not exhibit monopsonistic dynamics, which have been used before to explain Card and Krueger (1993).

Figure 1: Preference heterogeneity and low-skill employment in F

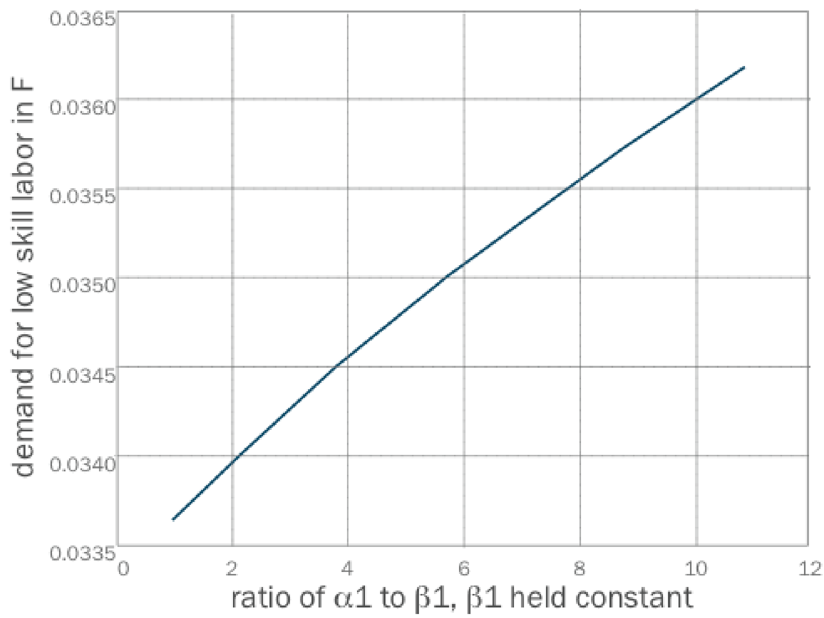


Figure 2: Elasticity of substitution and low-skill employment in F

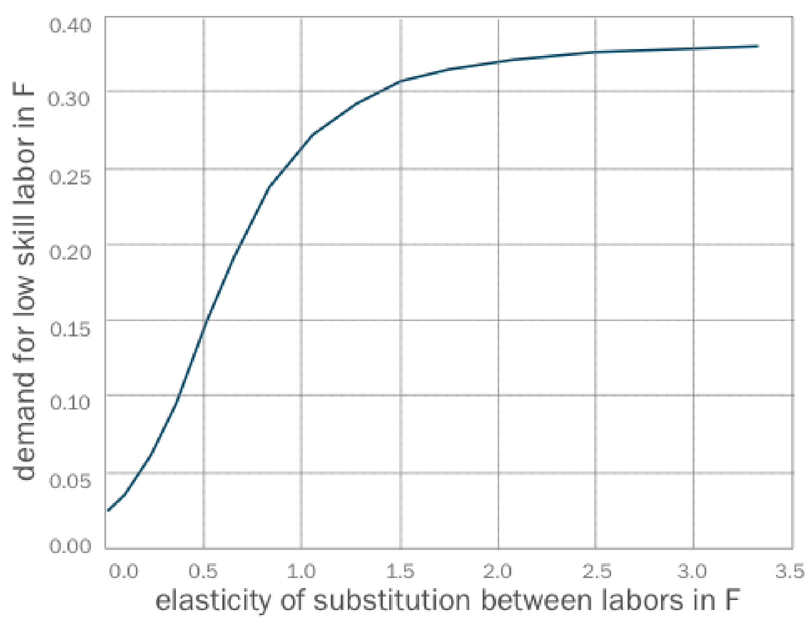


Figure 3: Transfer size and low-skill employment in F

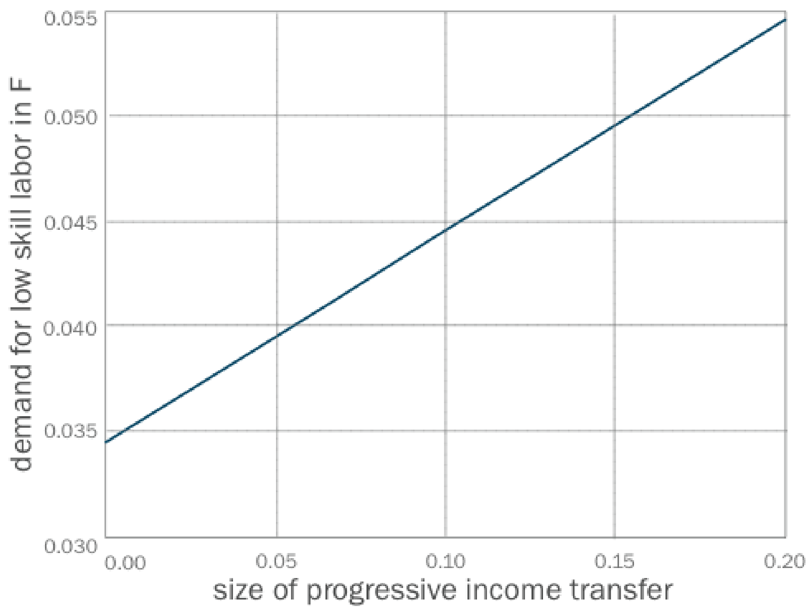
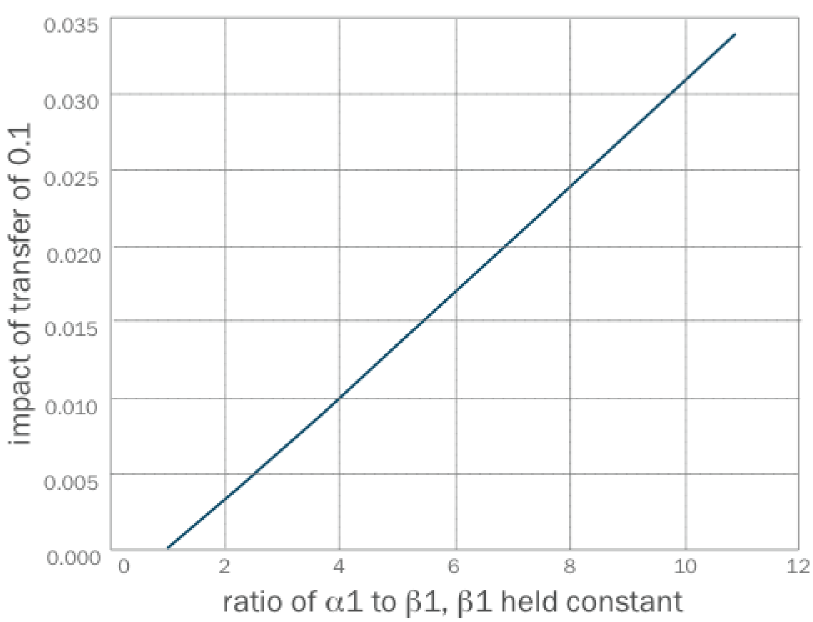


Figure 4: Preference heterogeneity and transfer impact on F low-skill labor



4.2 Progressive Income Transfers

The next two graphs reflect impacts on low-skill labor demand in the F sector when a progressive income transfer I is present in the economy. In this case, I is taken from consumer 2 and given directly to consumer 1. Figures 3 and 4 illustrate how equilibrium behaves with this inclusion. Figure 3 illustrates what happens to low-skill labor employment in the fast-food sector when the size of the progressive income transfer increases – the dependent variable increases linearly with the transfer size, as the transfer size increases demand for fast food at a linear rate as well. Because the low-skilled consumer spends a larger proportion of income on F, a lump-sum transfer from the high to low-skilled consumer results in the market demand for F increasing by $I(\alpha_1 - \beta_1)$. To satisfy greater demand for F, the F sector employs more low-skilled labor. More interesting is when the progressive income transfer is held at a given size $I = .1$ and I observe by how much it increases low-skill labor employment in the fast-food sector when I alter the degree of preference heterogeneity. The x-axis of the graph is the same as in figure 1, but the y-axis now measures the difference in employment that the transfer causes. In this case, the transfer's impact increases exponentially as the ratio of α_1 to β_1 is driven up. The greater the heterogeneity in preferences, the larger the demand increase for F after a progressive income transfer. From this, one can see that heterogeneous preferences can result in at least some progressive income policy resulting in higher employment for low-skill workers in certain sectors.

4.3 Impact of Minimum Wages

Recall that T_l is the amount of units of labor that the low-skilled consumer can contribute to the economy. Thus, decreasing T_l effectively shifts the labor supply curve to the left, resulting in a higher equilibrium low-skill wage. This is how I introduce minimum wages into the economy. I start with $T_l = 1$ for simplicity and find that a reduction of T_l to .95 generally results in an approximately 20% increase in the low-skill wage, which is the proportion that I am interested in investigating.

Figures 5 and 6 deal with the welfare impacts of minimum wage shifts, measured by percentage increase or decrease in consumption of E. Welfare impacts for the low-skill workers are positive; this is to be expected, as their income increases after the minimum wage increase, and they are able to consume more through essentially acting as a monopolist with their labor. The arc-like shape of the curve suggests two competing pressures in how welfare is impacted depending on the elasticity of substitution between inputs in the F sector. As the elasticity of substitution decreases, the producer is less capable of moving away from low-skill labor, which results in the price of F increasing in order to fulfill zero-profit conditions.

Figure 5: Ratio of elasticities of substitution and low-skill welfare impacts

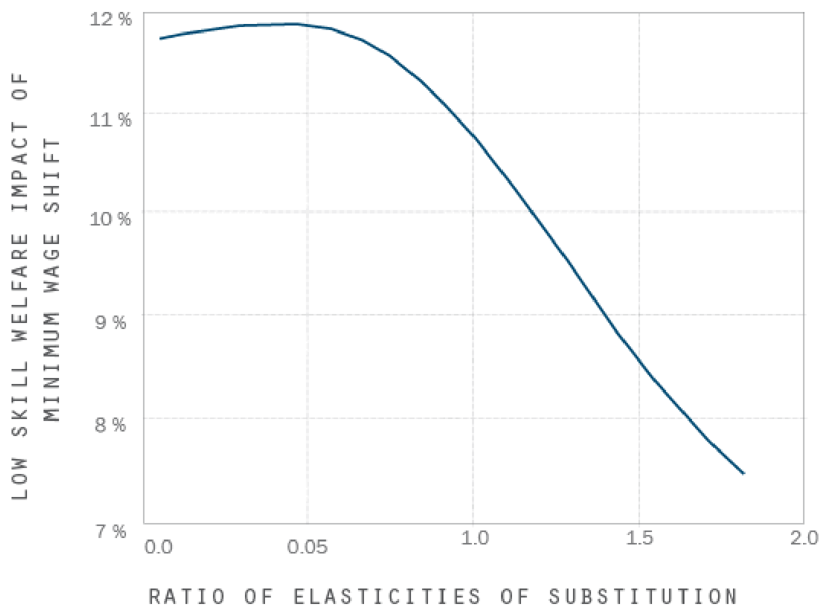


Figure 6: Ratio of elasticities of substitution and total welfare impacts

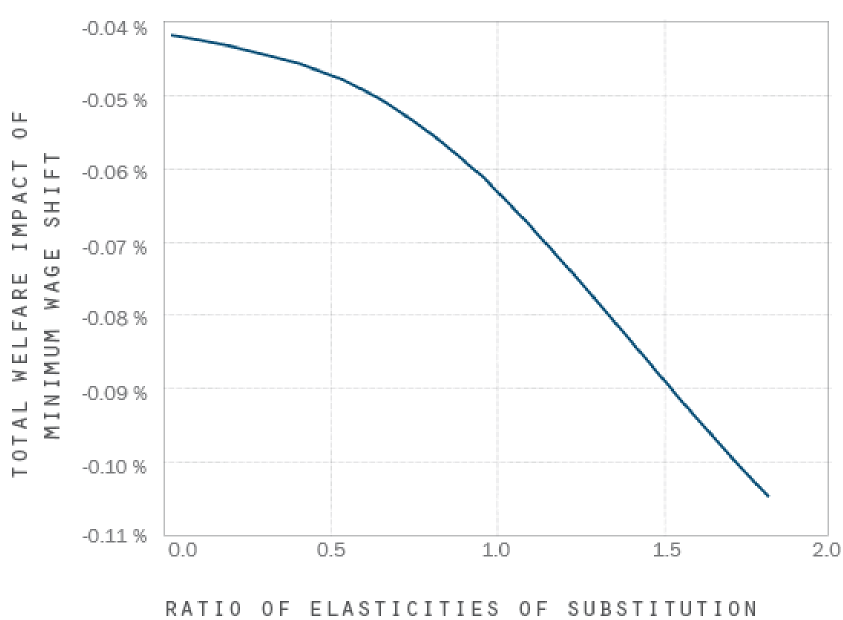
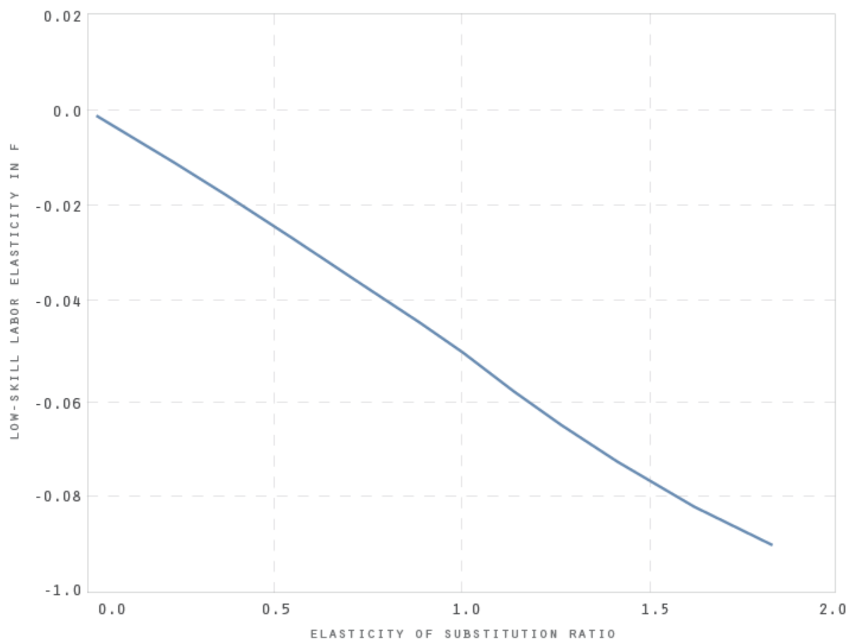


Figure 7: Ratio of elasticities of substitutions and low-skill elasticity in F



However, as the firm becomes too elastic between inputs, it can move away from low-skill labor employment to a degree that the wage increase can barely offset. Unsurprisingly, total welfare impacts indicate deadweight loss, measured by percentage change in aggregate output following the minimum wage shift. When I weight welfare changes based on the proportion of individuals in the economy who are either high or low skill, the welfare decrease among the high-skilled individuals that comes from a decrease in their real wage offsets the welfare increase of the low-skilled individuals. However, as the shifts in real wages and prices are quite modest, total welfare impacts overall remain small. Output in the F sector alone may increase when the F sector is highly inelastic, but deadweight loss is always present in the E industry.

However, the most important results of the model reside in figure 7, which documents the low-skill employment impacts of a minimum wage shift in the F sector. The y-axis represents the elasticity of low-skill labor in F, or the percentage change in employment over the percentage change in low-skill wage. The x-axis plots the ratio of the elasticity of substitution of the F firm to the E firm with the E firm's constant of elasticity ρ_e held constant. ρ_f is started at highly inelastic values and increased gradually. As seen from the graph, the elasticity of labor in response to a minimum wage shift may actually approach positive levels given a fast-food sector that is sufficiently unable to substitute away from low-skill labor. It is conceivable that this may be the case in the real world; fast-food franchises are likely unable to attract high-skill labor and need fixed amount of labor to operate the machines in the venues. As such, the locations may be relatively unable to adapt to minimum wage shifts and could have to increase employment to adapt to increased demand for their food. However, employment in the E industry invariably decreases, which is unsurprising given that the two types of consumers spend identical shares of income on it. The differences in income expenditure shares exist only in the amount of F and leisure consumed – both types of consumers spend half their income in E.

The results indicate that a sufficiently low elasticity of substitution in the fast-food sector can result in near-zero employment effects as a result of the introduction of a minimum wage. The timing of these results is worth discussing. As the model used in this simulation only occurs over one period, the results predicted are those that may be observed in the short run. Longer-run theoretical minimum wage impacts will be discussed later. One can easily conceive a story in which fast food franchises face great difficulties in substituting away from low-skill labor in the short run. Such businesses would be hard-pressed to attract high-skill laborers and likely need a minimum amount of workers to operate the capital in the establishment. Thus, the short run, fast-food venues may be unable to change hiring policies in response to low-skill wage increases, which is precisely what Card and Krueger (1993) seem to observe. These results are robust to a range of different parameterizations. Decreasing the elasticity of substitution in the E industry only makes the results more dramatic, and making the E industry more elastic to an extent does not change the result of potential ambiguity in low-skill employment in F following a minimum wage shock. Increasing β_1 does reduce the effect – for instance, if β_1 is raised to .04, then there are no demand changes to F after the minimum wage shift, and the increase of low-skill wage only results in employment decreasing for the poorer individual. However, the general theme of the results hold for any $\beta_1 < \alpha_1$. This gives the intuition that assessing preference heterogeneity may be useful in predicting employment impacts from minimum wages for low-skilled laborers in specific sectors.

4.4 Addition of Liquidity Constraints

There is strong evidence that individuals in lower income groups face strong liquidity constraints. In general, individuals of lower income have the lowest ratio of assets to desired borrowing levels, resulting in borrowing constraints. Furthermore, data from the consumer expenditure survey indicate that food, clothing, utilities, health care, housing and transportation costs make up roughly 85% of expenditure for households with income between $15 and $20 thousand per year. The strongest indicator of liquidity constraints, however, comes from the observation that these same households dedicate merely 2.5% of yearly income to saving, which may indicate that they have very little extra income to work with. Thus, the inclusion of liquidity constraints for the lower-income group is consistent with the data. Following this, one would expect a higher proportion of exogenous income to be spent on non-essentials, such as fast-food. Normally, simulating liquidity constraint effects requires an economy with multiple periods and income uncertainty. Myour model takes place in only one period, so such tools are not available to us. To simulate liquidity constraint effects, I do the following. I first calculate the amount of income the low-skilled consumer receives before the minimum wage shift. I then calculate income received after the minimum wage shift. Subtracting the first from the second produces a measure of how much income increases from the minimum wage shift. Recall that, before the minimum wage shift, I assume that the consumer spends 4% of his or her income on fast food. I now assume that a set percentage of the new money that comes in after the minimum wage shift is spent on fast food as well and that this percentage is greater than 4%. Set this proportion of new money spent on fast food to be $\gamma > \alpha_1$ To find the money spent on fast food after the minimum wage shift by the low-wage consumer, I take the sum

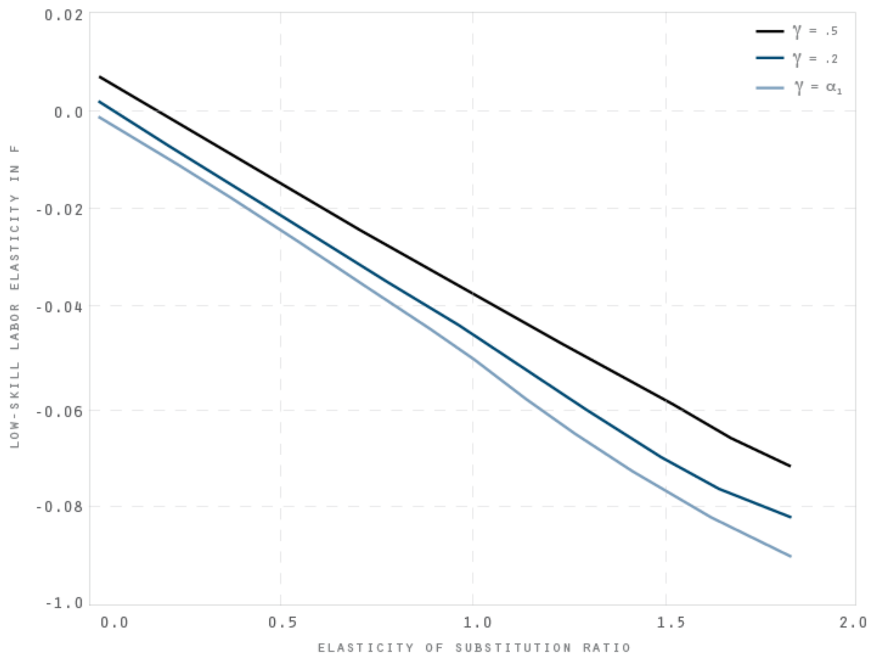
$$(\alpha_1 * w_l^0 * T_l^0) + (\gamma * (w_l^1 * T_l^1 - w_l^0 * T_l^0)),$$

where w_l^0 and T_l^0 are wages and hours before the minimum wage shift and w_l^1 and T_l^1 are symmetric parameters for after the minimum wage shift. I then divide this sum by total income earned after the minimum wage shift to find the new share of income dedicated to fast food. This new share can then be used in the Marshallian demand function in place of the original .04 measure to determine demand for fast food from the low-income group.

4.5 Results of Liquidity Constraint Addition

The inclusion of liquidity constraints for the low-skilled consumers augments the results and increases the low-skill labor elasticity in F in response to a minimum wage shift. Thus, liquidity constraints may be very important in the short-term effects of minimum wage policy. Figure 8 plots the elasticity of labor in the F sector given γ values of .2 and .5 and compares the results to the original value of $\gamma = \alpha_1$. The higher the γ, the more elastic the F industry can be between low and high-skill labor while maintaining the result that an increase in minimum wage results in higher low-skill labor in F. With sufficiently high liquidity constraints and low elasticity, employment effects may even be positive. The intuition follows a similar thread to earlier results - with higher preferences for spending exogenous income boosts on F, the minimum wage increases result in even higher demand for F in the presence of liquidity constraints. To fulfill this demand, the F sector must employ more labor – how much of that labor is high and low-skill depends on

Figure 8: Low-skill elasticity in F with liquidity constraints



the elasticity between the inputs. Thus addition provides further theoretical motivation behind the results that Card and Krueger (1993) observe.

Thus, a set of reasonable assumptions: liquidity constrained low-income consumers who spend a larger proportion of their income on fast food than the wealthy, and a fast food industry that faces short-run difficulties in reducing low-skilled employment, results in ambiguous or positive employment impacts in the fast food industry following a minimum wage increase. However, as evidenced by Figure 3, direct income transfers may be a more direct way to increase low-skill employment from a standpoint of policy comparison.

5 The Hopenhayn Model

The previous model is interesting in analyzing how minimum wage policy can impact low-skill laborers. However, due to being a static economy, the model lacks power in predicting long-run consequences of economic policy. It would also be worthwhile to investigate how minimum wages may impact long-run firm functioning and entry and exit through the use of a dynamic model. Hopenhayn (1992) develops a dynamic stochastic model for a competitive industry that has impressive power in analyzing the possible efficiency consequences of certain economic and market policy, especially when focusing on firm entry and exit. While his paper was not the first to introduce idiosyncratic productivity shocks to firms in every period, Hopenhayn's model possesses market advantages in its simplicity and its introduction of concept of stationary equilibrium. In this exercise, I utilize and adapt Hopenhayn's model to evaluate minimum wage policy - to my knowledge, this has not been done before. Together with the prior section, this analysis provides motivation for formerly puzzling empirical results.

5.1 Initial Construction

My construction of the model follows a similar framework to Hopenhayn and Rogerson (1993). In their paper, the authors use Hopenhayn's model to estimate the efficiency impacts of increased firing costs, finding that their effects are strongly negative.

Every firm in the economy has a stochastic production function using labor as its sole input. In period t, if a firm employs l_t low-skilled workers and h_t high-skilled workers with output price p_t, profits for period t are

$$p_t f(l_t, h_t, s_t) - w_l l_t - h_t - p_t c_f$$

Many of these terms deserve some explanation. w_l is the wage rate for low-skilled workers; the wage for high-skilled workers is normalized to be one. The term s_t is a firm-specific shock to production opportunities in period t. Identical to Hopenhayn and Rogerson (1993), the shock takes values in R_+ and follows a first-order Markov process described by a function $F(s, s')$, with the interpretation that for each current value of the shock, denoted by $s, F(s, \cdot)$ is the distribution function for the next period's value of the shock, denoted by s'. The shocks that every firm faces are independent of the shocks the other firms face, but all shocks evolve according to F. Finally, c_f is a fixed operating cost (denominated in units of output) incurred by the firm in each period in which it remains in the market - the inclusion of a fixed operating cost makes for there to be a meaningful possibility of firm exit.

Now consider the decisions made by each firm and the timing of them. The period t decision is made by a firm that was in operation in period $t-1$, at which time the firm faced a productivity shock equal to s_{t-1} and employed l_{t-1} low-skilled workers and h_{t-1} high-skilled workers at appropriate wages. At the start of period t, the firm must decide whether to stay in or exit the market - if the firm exits, they receive no revenue, but do not have to pay wages and do not have to endure the fixed cost c_f. If it stays, the firm automatically incurs the fixed cost c_f and will subsequently discover what s_t turns out to be. The firm then chooses low and high-skilled employment, produces output, and sells it at the period t price, with this process repeated in every period after this in which the firm still stays in the market. Firms will thus choose to exit when their prospects look sufficiently poor and they do not want to pay the fixed cost - this based on the shock s_{t-1} they observed before. Were there no fixed cost, the firm could simply wait for better times, possibly producing zero during the wait. (i.e. a higher realization of s).

Potential market entrants face a similar situation. Assume that there is a continuum of potential entrants in each period. Those who do enter incur a one-time cost of c_e, denominated in units of output. After this fixed cost has been paid, entrants are now in the same position as incumbent firms and receives its initial value of s as a draw from the distribution v. Draws are independent of one another and, of course, evolve according to F following the initial period. All firms behave so as to maximize the expected discounted present value of profits.

5.2 Preferences

Assume that all low-skilled workers are a continuum of identical agents distributed uniformly over the unit interval with preferences defined by

$$\sum_{t=1}^{\infty} \beta_1^t [u_1(c_t) - v_1(l_t)],$$

where c_t and l_t are consumption and labor supply in period t. Similarly, high-skill laborers are a continuum of identical agents distribued uniformly over the unit interval with preferences defined by

$$\sum_{t=1}^{\infty} \beta_2^t [u_2(c_t) - v_2(h_t)],$$

Consumption is non-negative, and labor supply is restricted to being either zero or one, so as to ensure that the number of employees at a firm is well-defined. Further, assume that low-skill and high-skill laborers enter low-skill and high-skill employment lotteries and have access to markets to diversify idiosyncratic risk. This, following from Rogerson (1988), will ensure that the mass of low and high-skill laborers will behave as two representative agents with preferences defined by

$$\sum_{t=1}^{\infty} \beta_1^t [u_1(c_t) - a_1 L_t],$$

$$\sum_{t=1}^{\infty} \beta_2^t [u_2(c_t) - a_2 H_t],$$

Where L_t and H_t are the fraction of low and high-skill workers employed in period t (see Appendix for a detailed explanation of this conclusion). Ownership of firm profits are divided up between the masses of low and high-skilled laborers based on the wages of low and high-skill workers, as wages are assumed to be indicative of relative productivity levels. Thus, low-skill laborers receive $\frac{w_l}{w_l+1}$ of profits while the high-skill laborers receive $\frac{1}{w_l+1}$ of profits. Within the groups, ownership of the firms is divided up evenly.

5.3 Optimal Behavior

Consider a firm that was in business last period, employing l and h low and high-skill workers and receiving a shock s. The value equation for the firm is

$$W(s;p) = \max_{l,h\geq 0; x\in\{0,1\}} pf(l,h,s) - w_l l - h - pc_f + \beta(1-x)[E_s W(s';p)].$$

where E_s denotes expectations conditional on the current value of s, and s' denotes the next (and random) value of s. I assume p to be stationary for the time being. If the value function W is known, the value of entering gross of entry costs, W^e, can be found with

$$W^e(p) = \int W(s;p)dv(s)$$

The decision problem will produce three rules: one for the optimal choices of employment for low and high-skill labor, and the other for the correct stay/exit decision at the beginning of the next period. I express these as $L(s;p)$, $H(s;p)$, and $X(s;p)$. X can take the values of 1 and 0, with 1 indicating the decision to exit and 0 the choice to stay.

5.4 Industry Dynamics

With no adjustment costs, the state of an individual firm is described by the pair (s;p), and the state of the economy is given by the distribution of the state variables for all firms. This can be naturally expressed as a measure over $(s;p)$, which I denote as $\mu(s;p)$.

This information is sufficient to map the evolution of the economy over time as long as prices are constant. After incumbent firms make their stay/exit decisions, let incumbents who are staying be summarized by a measure μ, and let the mass of firms that enter be described by the measure M. Firms follow optimal employment and entry/exit rules, and new information regarding shocks is revealed to the remaining incumbent firms - the new state of the incumbents will be denoted by μ'. The transition from μ to μ' will be given by $\mu' = T(\mu, M; p)$. Price enters into the equation because it influences entry and exit decisions. T is linearly homogenous in μ and M jointly, but not in μ alone; e.g. if the economy begins with twice as many firms of each type, it does not end with twice as many of each type one period later unless entry is doubled again.

5.4.1 Definition of Aggregates

Output in a given period, Y, as a function of the variables μ, M, and p can be expressed as

$$Y(\mu, M; p) = \int [f(L(s;p), (H(s;p), s) - c_f] d\mu(s) \\ + M \int (f(L(s;p), (H(s;p)), s) dv(s)$$

The first integral is created using optimal employment rules and integrated over the distribution of incumbents. The second does the same for new entrants, but all have a value of 0 for last period's employment, and the distribution of idiosyncratic shocks is given by v.

Furthermore, low and high-skilled labor demands as well as profits are given by

$$D^l(\mu, M, p) = \int L(s;p) d\mu(s) + M \int L(s;p) dv(s)$$

$$D^h(\mu, M, p) = \int H(s;p) d\mu(s) + M \int H(s;p) dv(s)$$

$$\Pi(\mu, M, p) = pY(\mu, M, p) - w_l D^l(\mu, M, p) - D^h(\mu, M, p) - Mpc_e$$

5.4.2 Definition of Equilibrium

DEFINITION: A stationary equilibrium consists of an output price $p^* \geq 0$, a mass of entrants $M^* \geq 0$, a preference parameter A_1 and a measure of incumbents μ^* such that:

$$D^l(\mu^*, M^*, p^*) = S^l(p^*, \Pi(\mu^*, M^*, p^*), A_1)$$

$$D^h(\mu^*, M^*, p^*) = S^h(p^*, \Pi(\mu^*, M^*, p^*))$$

$$T(\mu^*, M^*, p^*) = \mu^*$$

$$W^e(p^*) \leq p^* c_e$$

Written out, the equilibrium must entail that demand equals supply in the low and high-skill labor market, that the state of the economy must be such that optimal firm behavior results in a repetition of the state in the next period, and that entering firms must be willing to enter. Strict equality holds in the fourth case if there are a positive number of entrants in the economy, which is the case that I am interested in investigating.

5.5 Programming Procedure

The model is solved in four steps. Assuming positive entry, the first step uses the fourth equilibrium condition to determine the price level p^* in equilibrium. The second step uses the transition function T and finds a pair (μ^*, M^*) such that μ^* is a fixed point of the transition function $T(\mu, M^*, p^*)$. This process is made easier through initializing a first guess of M to be $M = 1$. After finding the fixed point of $T(\mu, 1, p^*)$, denoted by $\hat{\mu}$ I can solve for the level of entrants and exiters that satisfies the labor market-clearing conditions. Because T is linearly homogenous, it follows that if there were M entrants instead of 1 then $M\hat{\mu}$ would be the fixed point of the transition matrix. Thus, I then choose the scale factor M^* such that the high-skill labor market clearing conditions are also satisfied. Finally, given the number of incumbents and entrants that satisfies the high-skill labor market, there must also be a set demand for low-skill labor. In the case that there is a stationary equilibrium to begin with, the addition of a minimum wage will result in the labor market for low-skill labor not clearing due to a labor surplus. Most industry statistics remain computable in this case, but if I wanted to compute welfare among the low-skill workers, I would have to force them to consume more leisure than they would want.

5.6 Specification and Calibration

With aggregates and equilibrium defined, I move to specifying the functions:

$$u_1(c_t) = u_2(c_t) = \ln(c_t);$$

$$v_1(L_t) = A_1 L_t,\ v_2(H_t) = A_2 H_t;$$

$$f(l, h, s) = s(\delta l^\rho + (1-\delta)H^\rho)^{\theta/\rho},\ 0 \le \theta < 1.$$

With no labor adjustment costs in the economy, one can readily define optimal behavioral rules for the firm:

$$L(s;p) = (sp\theta\delta(1-\delta))^{-1/(\theta-1)}(w_l(1-\delta))^{1/\rho-1} \left(\delta(w_l(1-\delta))^{(\rho/\rho-1)} + (1-\delta)(w_h\delta)^{(\rho/\rho-1)}\right)^{(\rho-\theta)/\rho(\theta-1)};$$

$$H(s;p) = (sp\theta\delta(1-\delta))^{-1/(\theta-1)}(\delta)^{1/\rho-1} \left(\delta(w_l(1-\delta))^{(\rho/\rho-1)} + (1-\delta)(w_h\delta)^{(\rho/\rho-1)}\right)^{(\rho-\theta)/\rho(\theta-1)};$$

$$X(s;p) = 1 \text{ for some sufficiently low } s.$$

Furthermore, I define optimal behavior from the representative consumers. Defining Π to be aggregate profits in the economy given a measure of incumbents μ and entrants M, low and high-skill supply can be found with

$$L_t = 1/A_1 - \Pi \frac{w_l}{w_l + 1};$$

$$H_t = 1/A_2 - \Pi \frac{1}{w_l + 1}.$$

With optimal behavior defined, I calibrate the economy. Much of these calibrations were devised by Professor Dean Corbae (private communication) to match data moments. I set $\theta = .64$ and $\beta = .8$ as the parameters that ensure decreasing returns to scale and future discounting. I also define $A_2 = 1/200$ and the fixed costs of operating and entering $c_f = 10$ and $c_f = 15$ units of output. The most important calibration involves the possible shocks and the transitions between them. Define the set of shocks $s \in \{3.98e^4, 3.58, 6.82, 12.18, 18.79\}$, and define the transition matrix between shocks to be:

$$F(s'|s) = \begin{bmatrix} 0.6598 & 0.2600 & 0.0416 & 0.0331 & 0.0055 \\ 0.1997 & 0.7201 & 0.0420 & 0.0326 & 0.0056 \\ 0.2000 & 0.2000 & 0.5555 & 0.0344 & 0.0101 \\ 0.2000 & 0.2000 & 0.2502 & 0.3397 & 0.0101 \\ 0.2000 & 0.2000 & 0.2500 & 0.3400 & 0.0100 \end{bmatrix}$$

This matrix produces an invariant distribution, which I take to be the entrant distribution, given by

$$v(s) = \{0.37, 0.4631, 0.1102, 0.0504, 0.0063\}.$$

These distributions were calibrated to match the 37% exit rate reported in Hopenhayn and Rogerson (1993). In the calibration, firms exit upon receiving the lowest shock and stay in production at all other levels of operation. The wage of the high-skilled laborers is normalized to 1, and low-skill wage is moved from .5 to .6 to simulate a 20% increase in minimum wage.

5.7 Results

The proportion of firms of each size in this economy is roughly as follows:

$$\{0.24, 0.55, 0.15, 0.05, 0.007\}.$$

Table 1 documents changes in industry statistics following the 20% increase in minimum wage. As the statistics indicate, the minimum wage decreases the number of entrants in the economy, leading to fewer firms operating.

Table 1: Industry Statistics Before and After Minimum Wage Shift

	No Min. Wage	**With Min. Wage**
Output	2,564	2,487
Mean Output	4.01	4.15
Price	.345	.355
Entrants	154	144
Mean High-Skill Employment	1.97	2.09
Mean Low-Skill Employment	.175	.170
Total Employment	1,374	1,356
Firms Operating	638	598

However, mean output in the economy increases, indicating that the minimum wage induces a “survival of the fittest” dynamic in which only higher-functioning firms continue producing. The increase in minimum wage also raises equilibrium prices in the economy, which induces an incentive for greater production levels. However, because the cost of low-skill labor has increased, this induces producers to increase their usage of high-skill labor, evidenced by the increase in average high-skill employment following the minimum wage shift. However, average low-skill employment declines. These results are markedly different from those of Aaronson, French and Sorkin (2013), who find ambiguous employment impacts and increased entry and exit into and out of the economy. The key difference between my model and theirs is that producers in my model have more flexibility once they have entered the economy, which may entail lower initial exit and thus fewer entrants.

Table 2 provides a more in-depth review of the employment impacts of minimum wages through reporting optimal high and low-skill employment levels at every level of shock before and after the wage shift. Across all levels, low-skill wage decreases by roughly three percent and high-skill wage by approximately four percent. The intuition is that after the minimum wage shift equilibrium price increases, which entails greater revenue from produc-

Table 2: Optimal Employment Across Shock Levels with, w/o Min. Wage

	No Min. Wage		***With Min. Wage***	
Shock	**High-Skill**	**Low-Skill**	**High-Skill**	**Low-Skill**
1	0	0	0	0
2	.43	.038	.457	.037
3	2.577	.229	2.739	.222
4	12.906	1.147	13.71	1.113
5	43.04	3.825	45.73	3.71

tion. With high-skill labor being relatively cheaper, producers demand more of it and less of low-skill labor.

This analysis provides several interesting conclusions regarding the long-run impact of minimum wages. While employment impacts in the short run may be ambiguous or positive, as evidenced by the earlier model in this paper, the long-run impacts of minimum wages are negative in this dynamic model, as evidenced by the different stationary equilibrium after a minimum wage implementation. It should be acknowledged that this analysis does not provide any information regarding the transition path to the stationary equilibrium; in other words, the simulations do not report how periods it will take the economy to shift to the new equilibrium or dramatic the process will be. However, assuming that the stationary equilibrium is reached eventually, the long-run results of minimum wages will entail decreased economy output and low-skill employment, less desirable consequences than those observed in the two-good model.

6 Conclusion

This paper has developed multiple models that allow for a more sophisticated theoretical analysis of minimum wage efficiency than exists in the current literature. First, a model is created that allows us to explain the historically debated results of Card and Krueger (1993) through creating an economy with consumers that have heterogeneous preferences. This model demonstrates that, given a certain set of realistic assumptions, short-run minimum wage impacts on low-skilled labor in industries that rely on it may be ambiguous or positive. Furthermore, in order to observe longer-term impacts of minimum wages on firm functioning, I create a dynamic programming model with exogenous production shocks that tracks firm entry and exist levels, among other things, before and after a minimum wage shift. The evidence from the dynamic model indicates that minimum wages may be detrimental to low-skill employment and economy output in the long run.

The analysis of this paper could be extended in a variety of ways. It may be informative to directly compare minimum wage policy to a simulation of the EITC in the two-sector model to see how the policies compare in an economy with heterogeneous preferences. Furthermore, the addition of labor adjustment costs and the possible introduction of a second good in the Hopenhayn model would greatly expand the depth of the model as well as its ability to analyze minimum wages in particular. While it may be furthered, the analysis in this paper should prove useful in explaining some previously perplexing empirical properties of minimum wages and will hopefully contribute in the ongoing discussion surrounding this policy issue.

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8 Appendix

8.1 Representative agent rule from Rogerson (1988)

Assume the economy consists of a continuum of identical agents with two commodities: labor and output All activity takes place in a single period, and labor is used to produce output according to a diminishing marginal product of labor function $f(N)$. Each worker is endowed with one unit of time and all have an identical utility function

$$u(c) - v(n)$$

Because labor is indivisible, $n \in \{0, 1\}$. Assume that $v(0) = 0$ and $v(1) = m>0$. Define the consumption set X for each worker as

$$X = \{(c, n) \in R^2 : c \geq 0, n \in \{0, 1\}\}.$$

Now I include employment lotteries in the model. Define

$$X_1 = \{(c, n) \in X : n = 1\}$$

$$X_2 = \{(c, n) \in X : n = 0\}$$

$$\bar{X} = X_1 \text{ x } X_2 \text{ x } [0, 1]$$

$\bar{X}$ is the new consumption set for all workers. The set X_1 corresponds to the allocations wherein the individual supplies labor; X_2 represents the analogous case when no labor is supplied. The third element, a number in the interval $[0, 1]$, represents the likelihood that the X_1 allocation is realized. Thus, an element of $\bar{X}$ will be written $((c_1, 1), (c_2, 0), \phi)$, and the utility garnered from receiving the aforementioned allocation is

$$\phi[u(c_1) - m] + (1 - \phi)(u[c_2]))$$

And the utility maximization problem of the consumer becomes

$$\max_{c_1, c_2, k_1, k_2, \phi} \phi[u(c_1) - m] + (1 - \phi)[u(c_2)]$$

subject to

$$\phi c_1 + (1 - \phi)c_2 \leq w\phi$$

$$c_i \geq 0, \ i = 1, 2$$

$$0 \leq \phi \leq 1$$

Note that if $\phi \in (0, 1)$, then $c_1 = c_2$, as the first-order conditions for the above consumer problem are

$$\phi u'(c_1) = \phi\theta \text{ and } (1 - \phi)u'(c_2) = (1 - \phi)\theta$$

Where θ is the multiplier on the budget constraint. Also note that if $\phi \notin (0,1)$, there is loss of generality in having $c_1 = c_2$. So the consumer problem reduces to

$$\max_{c,\phi} u(c) - \phi m$$

subject to

$$c = w\phi + r,\ c \geq 0, 0 \leq \phi \leq 1$$

With this, equilibrium can be defined as a list (c, ϕ, N, w) such that

(i) (c, ϕ) solves the simplified consumer problem
(ii) (N) solves the profit maximization problem for producers
(iii) Markets clear, so $\phi = N$, $c = F(N)$

This equilibrium is identical to the equilibrium one would obtain for an economy with production function $f(N)$ with one agent whose utility is specified by $u(c) - mn$ with a consumption set identical to X but with divisible labor, so $0 \leq n \leq 1$. In the case of my model, $\phi = N$ represents the proportion of possible workers employed in the economy.